ANISOTROPY OF CREEP OF MATERIALS

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It has been shown that when ordinary stress-strain ($\sigma-\varepsilon$) diagrams are constructed for 20-mm thick rolled duralumin plate at various temperatures, the alloy behaves like an isotropic material, while a considerable degree of anisotropy is observed in creep testing.

A short review of studies of the anisotropy of creep of metallic and nonmetallic materials was given in [1], where concepts of the theory of plastic flow were used to demonstrate some possibilities of describing the anisotropy of creep by means of theoretical results in processing experimental data on the creep of plastics. The aim of the present investigation was to study, as in [2], the anisotropy of steady-state creep from the standpoint of the theory of viscous flow.

1. The specimens used in taking stress-strain diagrams were made from blanks cut in the plane of the rolled plate at 0°, 45°, and 90° to the rolling direction. The gauge portion of the tensile test pieces was 50 mm, their diameter 12 mm; specimens for compression tests were 8 mm in diameter. The tests were carried out at several constant temperatures. The measurements of the absolute strain and continuously increasing load in compression tests were recorded on motion-picture film and subsequently processed; in tensile tests, the load was increased in steps with the aid of weights and the strain gauges were read visually. One compression test lasted not more than 10 sec; it took 30–35 sec to carry out a tensile test. After plotting the stress-strain diagram, the cross section of each specimen was measured; in no case was any evidence of ovality found.

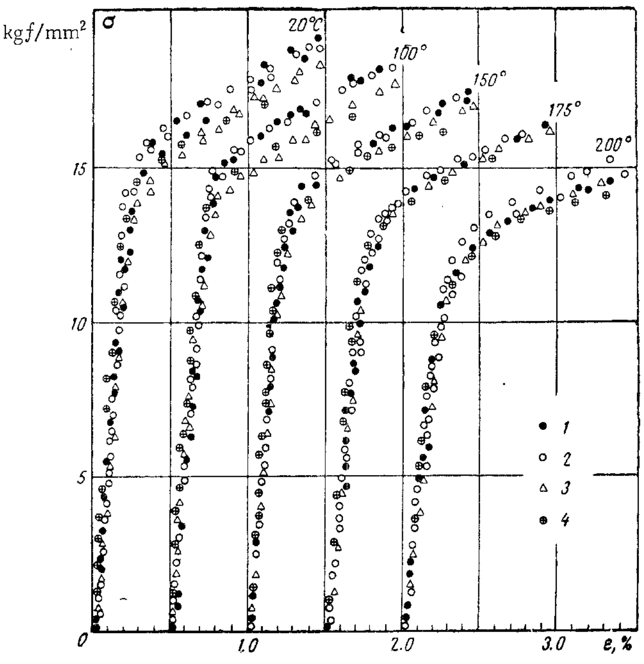

Fig. 1

The results of these tests are reproduced in Fig. 1, where black circles, open circles and open triangles relate to data obtained in compression for specimens at 0°, 45°, and 90° to the rolling direction, respectively; crosses relate to data for transverse tensile test pieces. It will be seen that all the points obtained at a given test temperature form a very narrow band not only in the elastic but also in the plastic range, from which it may be concluded that the material tested is isotropic with respect to its elastoplastic characteristics.

Quite different results were obtained in the creep tests. These were carried out on tensile specimens of the same shape and cut in the same directions relative to the direction of rolling as in the short-time tests. During each test measurements were taken of the axial elongation and dimensional changes of two mutually perpendicular specimen diameters, one of which was normal to the plane of the original plate. The transverse strains were measured with four microgauges mounted in diametrically opposite positions (on two mutually perpendicular diameters) on an invar ring which, secured to an elastic mount, was fitted round the heating furnace. The variation in specimen diameter was transmitted to the microgauges through quartz rods; the readings were taken visually or recorded with a motion-picture camera.

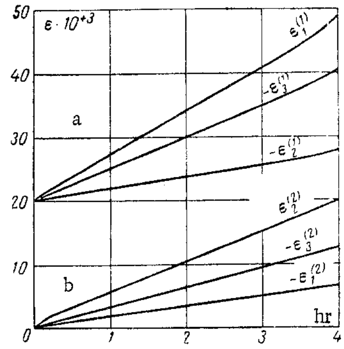

Fig. 2

The creep tests were carried out at 200°C and at stresses of 6–10 kg/mm²; creep curves constructed under these conditions showed practically no primary creep stage. The load either remained constant or was varied in stepwise fashion; in either case, the creep rates under equal loads were the same, and the creep rate deviated from a constant value only for a relatively short time when the load was increased. The sum of strains in three mutually perpendicular directions always approached zero, especially in the high stress range.

Creep curves obtained at σ = 8 kg/mm² are reproduced in Figure 2, where curves a and b relate to longitudinal and transverse tensile specimens and show the corresponding axial and radial strains. In both cases the rate of radial strain in a direction normal to the plane of the rolled plate was faster than the rate in a direction perpendicular to that direction, it being found that

$$\eta_{33}^{(1)} : \eta_{22}^{(1)} = k_1 \approx 2.7 \text{ — in the first case} \qquad (1.1)$$
$$\eta_{33}^{(2)} : \eta_{11}^{(2)} = k_2 \approx 2 \text{ — in the second case}.$$

Here, and subsequently, the indices denote directions: 1—the direction of rolling; 2—normal to the direction of rolling; 3)—normal to the plane of the rolled plate; the upper index indicates the direction of the applied load. Thus, for instance, $\eta_{33}^{(2)}$ denotes the rate of strain in the third direction under the influence of an axial load applied in the second direction.

To compare the behavior of the material studied in tension and compression, creep tests were carried out on cylindrical compression specimens (35 mm long, 12 mm diam.) cut in the same directions as for the tensile creep tests. The tests were carried out at the same temperature and in the same stress range, but only the axial strains were measured. The rates of creep in compression were in every case almost identical with the corresponding rates in tension, although the agreement was slightly worse in the high stress range.

The next series of compression tests was carried out on specimens of square cross section (10 X 10 X 19 mm) cut in the direction normal to the plane of the rolled plate; here again only the axial strains were measured. Taking into account the fact that the behavior of the material tested in the plane of the rolled plate is the same in tension and in compression, the possibility of extending this property to all the other directions was postulated.

The behavior of the material in creep was satisfactorily described by an equation of the flow theory type

$$\eta = B_i\sigma^n. \tag{1.2}$$

Here η is the creep strain rate, and $\dot{B}_1$ and n denote material constants. Processing of the experimental data gave the following values:

$$\begin{aligned}&\eta = 8 \text{ (for all stresses);}\\&B_1 = 5 \times 10^{-10}\ [\text{mm}^{2n}/\text{kg}^n\text{hr}] \text{ (in the direction of rolling);}\\&B_2 = 3 \times 10^{-10}\ [\text{mm}^{2n}/\text{kg}^n\text{hr}] \text{ (in the direction normal to rolling);}\\&B_3 = 17 \times 10^{-10}\ [\text{mm}^{2n}/\text{kg}^n\text{hr}] \text{ (in the direction normal to the plane of the rolled plate).}\end{aligned} \tag{1.3}$$

Thus, the properties of the material are characterized by orthotropism with quite a pronounced degree of anisotropy in crrep, in spite of the fact that the elastoplastic characteristics under short-time loading conditions are almost fully isotropic.

2. As previously stated, the main part of the creep strain under the experimental conditions employed (both at constant and stepwise increasing loads) is steady-state creep, and the process itself may be regarded as viscous flow with a nonlinear viscosity law. Ziegler [3], who used thermodynamic considerations in an attempt to prove the existence of creep potential for viscolinear media, showed that Onsager's theory may be hypothetically extended to the case of visco-nonlinear processes, when the creep potential is not a quadratic form of the variables. In the case of an isotropic body with visco-nonlinear flow, the creep potential should be taken in the form of a function of the stress tensor invariants or, in a simpler case, a function of a quadratic invariant, i. e., the stress intensity [4]. Extending this supposition to weakly anisotropic bodies, we assume that the creep potential is a function of a certain quadratic form, i. e., that the following relationship holds

$$\eta_{ij} = \frac{\partial\Phi(T)}{\partial\sigma_{ij}}, \quad T = a_{ijkl}\,\sigma_{ij}\sigma_{kl}. \tag{2.1}$$

Here η_{ij} are the creep strain rate tensor components, and T denotes a positive definite quadratic form which has 21 independent coefficients in the case of a body with arbitrary anisotropy and 18 coefficients in the case of a specially selected coordinate system [5].

For a uniaxial stress state Eq. (2.1) should coincide with Eq. (1.2), so that it is expedient to represent the potential function as a power function of the quadratic form

$$\Phi(T) = T^{(n+1)/2}. \tag{2.2}$$

In the case of an orthotropic body, the number of coefficients of the quadratic form can be reduced to six by choosing a coordinate system which is congruent with the principal anisotropy axes. In fact, multiplying both sides of (2.1) by σ_{ij}, summing over the indices, and taking into account (2.2), we obtain

$$\sigma_{ij}\eta_{ij} = (n+1)\,T^{(n+1)/2}, \tag{2.3}$$

i. e., in the stress space the specific power of energy dissipation in creep coincides (correct to a constant) with the creep potential. It is evident that with the given system of applied loads the specific power of energy dissipation does not depend on the choice of the coordinate system. Reversing in turn the direction of each of the coordinate axes, it can be easily demonstrated that the number of coefficients is reduced to nine, and that the quadratic form becomes

$$\begin{aligned}T(\sigma_{ij}) = {} & a_{1111}\sigma_{11}^2 + a_{2222}\sigma_{22}^2 + a_{3333}\sigma_{33}^2 + 2a_{1122}\sigma_{11}\sigma_{22} + \\ & + 2a_{1133}\sigma_{11}\sigma_{33} + 2a_{2233}\sigma_{22}\sigma_{33} + 4n_{1212}\sigma_{12}^2 + 4a_{1313}\sigma_{13}^2 + 4a_{2323}\sigma_{23}^2.\end{aligned} \tag{2.4}$$

As stated above, experiment supports the hypothesis of incompressibility of materials in creep, but then from (2.1) and (2.4) we have

$$\begin{aligned}\eta_{11} &= (n+1)\,T^{(n-1)/2}\,(a_{1111}\sigma_{11} + a_{1122}\sigma_{22} + a_{1133}\sigma_{33}),\\ \eta_{22} &= (n+1)\,T^{(n-1)/2}\,(a_{2211}\sigma_{11} + a_{2222}\sigma_{22} + a_{2233}\sigma_{33}),\\ \eta_{33} &= (n+1)\,T^{(n-1)/2}\,(a_{3311}\sigma_{11} + a_{3322}\sigma_{22} + a_{3333}\sigma_{33}).\end{aligned}$$

Hence it follows that

$$\begin{aligned}\eta_{11} + \eta_{22} + \eta_{33} = {} & (n+1)\,T^{(n-1)/2}\,[(a_{1111} + a_{2211} + a_{3311})\,\sigma_{11} + \\ & + (a_{1122} + a_{2222} + a_{3322})\,\sigma_{22} + (a_{1133} + a_{2233} + a_{3333})\,\sigma_{33}] \equiv 0.\end{aligned}$$

This equality should be identically satisfied for any values of σ_{ij} and $T(\sigma_{ij}) \neq 0$; hence there follow the three additional conditions

$$\begin{aligned}(a_{1111} + a_{2211} + a_{3311}) &= (a_{1122} + a_{2222} + a_{3322}) = \\ &= (a_{1133} + a_{2233} + a_{3333}) = 0,\end{aligned}$$

and the quadratic form is represented by an expression previously used by Hill to describe anisotropic plasticity

$$\begin{aligned}T(\sigma_{ij}) = {} & A_{11}(\sigma_{22} - \sigma_{33})^2 + A_{22}(\sigma_{33} - \sigma_{11})^2 + \\ & + A_{33}(\sigma_{11} - \sigma_{22})^2 + 2A_{12}\sigma_{12}^2 + 2A_{23}\sigma_{23}^2 + 2A_{31}\sigma_{31}^2,\end{aligned} \tag{2.5}$$

$$(A_{11} = -a_{2233},\ A_{22} = -a_{1133},\ A_{33} = -a_{2211},$$
$$A_{12} = 2a_{1212},\ A_{23} = 2a_{2323},\ A_{31} = 2a_{3131}).$$

The remaining six coefficients are determined from the results of creep tests under uniaxial stress state conditions. Let us assume that the characteristics of univariate creep in the three principal directions, i. e., B_1, B_2, and B_3, are known. If, taking into account (2.2) and (2.5), each of the normal stresses (in the appropriate principal direction) in (2.1) is successively assumed to be nonzero, if the expression obtained is equated to (1.2) with the corresponding value of B_1, and if then the terms associated with σ^n are equated, a system of three equations for A_{11}, A_{22}, and A_{33} is obtained

$$A_{22} + A_{33} = \left(\frac{B_1}{n+1}\right)^{2/(n+1)}, \quad A_{33} + A_{11} = \left(\frac{B_2}{n+1}\right)^{2/(n+1)},$$
$$A_{11} + A_{22} = \left(\frac{B_3}{n+1}\right)^{2/(n+1)}. \tag{2.6}$$

Solving these equations, we find

$$2A_{11} = \left(\frac{B_2}{n+1}\right)^{2/(n+1)} + \left(\frac{B_3}{n+1}\right)^{2/(n+1)} - \left(\frac{B_1}{n+1}\right)^{2/(n+1)}. \tag{2.7}$$

The other two coefficients are obtained by cyclic permutation of the indices.

For the material under consideration, substituting the values of B and n from (1.3), we obtain

$$A_{11} = 0.53\cdot 10^{-20/9}, \qquad A_{22} = 0.62\cdot 10^{-20/9},$$
$$A_{33} = 0.25\cdot 10^{-20/9}\ [\text{mm}^{2n}/\text{kg}^n\,\text{hr}]^{2(n+1)}.$$

If no experiments were carried out in the third direction, and if transverse strains were measured during tests in, for instance, the first direction, then from (2.1), (2.2) and (2.3) we have

$$\begin{aligned}\eta_{33}^{(1)} &= -(n+1)\,[A_{22} + A_{33}]^{(n-1)/2}\,A_{22}\sigma^n,\\ \eta_{22}^{(1)} &= -(n+1)\,[A_{22} + A_{33}]^{(n-1)/2}\,A_{33}\sigma^n,\end{aligned} \tag{2.8}$$

the ratio $\eta_{33}^{(1)} : \eta_{22}^{(1)} = k_1$ being known from experiment.

Using the first two equations in (2.6), we find

$$A_{22} = \frac{k_1}{1+k_1}\left(\frac{B_1}{n+1}\right)^{2/(n+1)}, \quad A_{33} = \frac{1}{1+k_1}\left(\frac{B_1}{n+1}\right)^{2/(n+1)},$$
$$A_{11} = \left(\frac{B_2}{n+1}\right)^{2/(n+1)} - \frac{1}{1+k_1}\left(\frac{B_1}{n+1}\right)^{2/(n+1)}. \tag{2.9}$$

The third equation in (2.6) makes it possible to determine B_3 from k_1 and known constants B_1 and B_2

$$B_3 = \left[B_2^{2/(n+1)} + \frac{k_1 - 1}{k_1 + 1}\,B_1^{2/(n+1)}\right]^{(n+1)/2}. \tag{2.10}$$

Similar relations can be obtained from experiments in the second direction; in particular, B_3 will be

$$B_3 = \left[B_1^{2/(n+1)} + \frac{k_2 - 1}{k_2 + 1}\,B_2^{2/(n+1)}\right]^{(n+1)/2}. \tag{2.11}$$

Substituting in (2.10) and (2.11) the appropriate values from (1.1) and (1.3), we find

$B_3 = 19 \times 10^{-10}$ (from experiments in the first direction);

$B_3 = 16 \times 10^{-10}$ (from experiments in the second direction).

Both these values are in good agreement with the value $B_3 = 17 \times 10^{-10}$ obtained from compression tests.

In a number of specific cases (e.g., a tube or a sphere under internal pressure, a rotating disc), when the problem is solved for the principal stresses and the coordinate system is congruent with the principal anisotropy axes, it is sufficient for the quadratic form to contain only the three coefficients A_{11}, A_{22}, and A_{33}, since the last three are not used in finding the solution.

To determine the remaining three coefficients in (2.5), it is necessary to know the characteristics of univariate creep in three directions which do not lie on one plane and which do not coincide with the principal directions. Let us, for the sake of simplicity, assume that these directions are in the three coordinate planes. Let us also introduce a new coordinate system obtained from the original by rotating axes X_1 and X_2 about axis X_3 through a positive angle of 45°. From experiments carried out in the direction X_1 we know B_{12} and n. Expressing the stress components (related to the old coordinate system which is congruent with the principal anisotropy axes) in terms of the stress σ_v (whose direction coincides with the axis X_1' of the new system)

$$\sigma_{11} = {}^1/_2\,\sigma_v, \qquad \sigma_{22} = {}^1/_2\,\sigma_v, \qquad \sigma_{12} = {}^1/_2\,\sigma_v,$$

and substituting their values in (2.1), we find, taking into account (2.2) and (2.5), the strain rate tensor components in the old coordinate system

$$\eta_{11} = \frac{n+1}{2^n}\,[A_{11} + A_{22} + 2A_{12}]^{(n-1)/2}\,A_{22}\sigma_v^{\,n},$$

$$\eta_{22} = \frac{n+1}{2^n}\,[A_{11} + A_{22} + 2A_{12}]^{(n-1)/2}\,A_{11}\sigma_v^{\,n},$$

$$\eta_{33} = -\frac{n+1}{2^n}\,[A_{11} + A_{22} + 2A_{12}]^{(n-1)/2}\,(A_{11} + A_{22})\,\sigma_v^{\,n},$$

$$\eta_{12} = \eta_{21} = \frac{n+1}{2^n}\,[A_{11} + A_{22} + 2A_{11}]^{(n-1)/2}\,A_{12}\sigma_v^{\,n}.$$

Performing an inverse transformation, we express the normal components of the strain rates in the new coordinate system in terms of the values of η_{ij} in the old system:

$$\begin{aligned}
\eta_{11}' &= \frac{n+1}{2^{n+1}}\,[A_{11} + A_{22} + 2A_{12}]^{(n-1)/2}\,(A_{11} + A_{22} + 2A_{12})\,\sigma_v^{\,n},\\
\eta_{22}' &= \frac{n+1}{2^{n+1}}\,[A_{11} + A_{22} + 2A_{12}]^{(n-1)/2}\,(A_{11} + A_{22} - 2A_{12})\,\sigma_v^{\,n},\\
\eta_{33}' &= -\frac{n+1}{2^{n+1}}\,[A_{11} + A_{22} + 2A_{12}]^{(n-1)/2}\,2\,(A_{11} + A_{22})\,\sigma_v^{\,n}.
\end{aligned} \tag{2.12}$$

Equating the expression for the first of these components to (1.2) and substituting the values B_{12}, we obtain (after eliminating σ_v^n) an equation

$$B_{12} = \frac{n+1}{2^{n+1}}\,[A_{11} + A_{22} + 2A_{12}]^{(n+1)/2} \tag{2.13}$$

from which the coefficient A_{12} is determined:

$$2A_{12} = 4\left(\frac{B_{12}}{n+1}\right)^{2/(n+1)} - A_{11} - A_{22}. \tag{2.14}$$

Rotating now the old coordinate system through 45° in the plane X_1X_3 without altering the position of X_2 and performing similar calculations, we obtain

$$B_{13} = \frac{n+1}{2^{n+1}}\,[A_{11} + A_{33} + 2A_{13}]^{(n+1)/2}, \tag{2.15}$$

from which we find A_{13}

$$2A_{13} = 4\left(\frac{B_{13}}{n+1}\right)^{2/(n+1)} - A_{11} - A_{33}. \tag{2.16}$$

Finally, performing a similar operation in the plane X_2X_3 and repeating the calculation, we find A_{23}:

$$2A_{23} = 4\left(\frac{B_{23}}{n+1}\right)^{2/(n+1)} - A_{22} - A_{33}. \tag{2.17}$$

Owing to technical difficulties, experiments in the direction not coinciding with one of the principal directions were carried out only in the plane X_1X_2 at 45° to the rolling direction. It was found that $B_{12} = 3.7 \times 10^{-10}$ and the ratio of transverse creep strains $\eta_{33}' : \eta_{22}' = k_{12} \approx 2.1$.

From (2.14), substituting the values B_{12}, A_{11}, and A_{22}, we find $A_{12} = 1.1 \times 10^{-20/9}$; the last two equations in (2.12) give

$$\eta_{33}' : \eta_{22}' = \frac{2\,(A_{11} + A_{22})}{2A_{12} - A_{11} - A_{22}} = 2.25,$$

which is in satisfactory agreement with the experimental value of k_{12}.

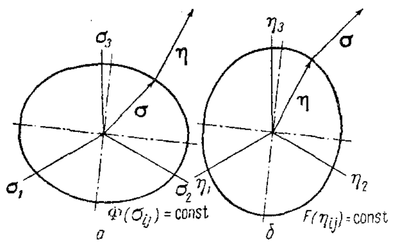

Fig. 3

From (2.1) we have

$$\begin{aligned}
2\frac{d\Phi}{dT}\sigma_{11} &= \frac{A_{11}}{\Delta}\eta_{11} + C, & 2\frac{d\Phi}{dT}\sigma_{12} &= \frac{1}{A_{12}}\eta_{12},\\
2\frac{d\Phi}{dT}\sigma_{22} &= \frac{A_{22}}{\Delta}\eta_{22} + C, & 2\frac{d\Phi}{dT}\sigma_{13} &= \frac{1}{A_{13}}\eta_{13},\\
2\frac{d\Phi}{dT}\sigma_{33} &= \frac{A_{33}}{\Delta}\eta_{33} + C, & 2\frac{d\Phi}{dT}\sigma_{23} &= \frac{1}{A_{23}}\eta_{23}.
\end{aligned} \tag{2.18}$$

Here, the value

$$\Delta \equiv A_{11}A_{22} + A_{22}A_{33} + A_{33}A_{11} \tag{2.19}$$

is larger than zero in view of the positive definiteness of the quadratic form (2.5), and C is an arbitrary constant appearing in the solution due to the linear dependence of the first three equations in (2.1). Expressing σ_{ij} from (2.18) in terms of $d\Phi/dT$ and η_{ij} and substituting in (2.5), we obtain (taking into account (2.2)) after some simple transformations, in view of incompressibility of the material,

$$\begin{aligned}
(n+1)^2\,T^n &= \frac{A_{11}}{\Delta}\eta_{11}^2 + \frac{A_{22}}{\Delta}\eta_{22}^2 + \frac{A_{33}}{\Delta}\eta_{33}^2 +\\
&+ \frac{2}{A_{12}}\eta_{12}^2 + \frac{2}{A_{23}}\eta_{23}^2 + \frac{2}{A_{31}}\eta_{31}^2 \equiv \Gamma,
\end{aligned} \tag{2.20}$$

i.e., the same functional dependence as that which in (1.2) relates the quadratic forms of the stresses and strain rates.

Equation (2.20) may be written in the form

$$(n+1)\,T^{(n+1)/2} = (n+1)^{-1/n}\,\Gamma^{(n+1)/2n}. \tag{2.21}$$

Comparison with (2.3) leads to the conclusion that the right side of (2.21) represents the specific power of energy dissipation in strain rate space. Taking the derivative of this expression, multiplying both sides by η_{ij} and summing, we obtain

$$\frac{(n+1)^{(n-1)/n}}{n}\,\Gamma^{(n+1)/2n} = Y_{ij}\eta_{ij}. \tag{2.22}$$

Hence, in view of (2.21), we conclude that Y_{ij} represents (correct to its factor) the stress tensor components, and the function

$$F(\eta_{ij}) = q\Gamma^{(n+1)/2n} \quad (q - \text{proportionality factor}) \tag{2.23}$$

by analogy with (2.2) may be called the stress potential function.

Figure 3 shows, in the deviatoric plane, the projections of the surfaces of constant specific power of energy dissipation in creep in: a) stress space Φ = const, and b) strain rate space F = const. Both surfaces are convex and represent, respectively, potential surfaces for: a) strain rates and b) stresses.

When the degree of anisotropy decreases and all B_1 approach a single value B_0, from (2.7), (2.14), (2.16), and (2.17) we obtain

$$3A_{11} = 3A_{22} = 3A_{33} = A_{12} = A_{23} = A_{31} = \frac{3}{2}\left(\frac{B_0}{n+1}\right)^{2/(n+1)}; \tag{2.24}$$

the quadratic forms T and Γ are transformed, respectively, into the second invariants of the stress and strain rate deviators, and the relation (2.20) becomes an expression describing the dependence of strain rate on stress intensity, which is often used in calculations relating to steady-state creep.

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